The details of the exact solutions for a bimaterial composite under uniaxial compression

The analytical equivalent resistance for the verification model presented in Fig. 1 is given by

$$\sum R_i = \sum \frac{\rho_i L_i}{A_i} = \rho_1 L_1/A_1 + 2\,\rho_2 L_2/A_2 \tag{A1}$$

where ρ is the electrical resistivity, L is the thickness in the direction of the applied compression, and A is the cross-sectional area normal to the load. The indices 1 and 2 denote the sensing layer and electrodes, respectively. With zero Poisson's ratio (ν) considered for the presented validation, only L varies with the loading, and therefore, the ratio of the resistances before (R_0) and after (R) deformation is given by

$$\frac{R}{R_0} = \frac{\rho_1 L_1(1-\varepsilon_1) + 2\rho_2 L_2(1-\varepsilon_2)}{\rho_1 L_1 + 2\rho_2 L_2} \tag{A2}$$

where ε is the magnitude of the normal strain. For the linear elastic model, the normal strain is given by P/E_i where P is the pressure and E is the modulus of elasticity. Therefore,

$$\frac{R}{R_0} = 1 - \frac{\rho_1 L_1/E_1 + 2\,\rho_2 L_2/E_2}{\rho_1 L_1 + 2\rho_2 L_2} P \tag{A3}$$

For the analytical validation of hyperelasticity, we considered a neo-Hookean model defined by a strain energy density function in the form of

$$W = \frac{\mu}{2}(\bar{I}_1 - 3) + \frac{\kappa}{2}(J-1)^2 \tag{A4}$$

where $\mu = E/2(1+\nu)$ is the shear modulus and $\kappa = E/3(1-2\nu)$ is the bulk modulus. In Eq. (A4), J is the third principal invariant of the right Cauchy-Green deformation tensor and $\bar{I}_1$ is the first invariant of the isochoric part of the deformation tensor:

$$J = \det \mathbf{F} = \lambda_1 \lambda_2 \lambda_3 \tag{A5}$$

$$\bar{I}_1 = (\det \mathbf{C})^{-\frac{1}{3}} \operatorname{tr} \mathbf{C} = J^{-\frac{2}{3}} \operatorname{tr} \mathbf{C} = J^{-\frac{2}{3}}(\lambda_1^2 + \lambda_2^2 + \lambda_3^2) \tag{A6}$$

where λ_1, λ_2, λ_3 are the principal stretches. The principal components of the Cauchy stress are $\sigma_i = (\lambda_i/J)\,\partial W/\partial\lambda_i$. Substituting Eq. (A4) results ($i = 1,2,3$)

$$\sigma_i = \mu\,(\lambda_i^2 - I_1/3)/J^{5/3} + \kappa(J-1) \tag{A7}$$

where $I_1 = \mathrm{tr}\,\mathbf{C} = \lambda_1^2 + \lambda_2^2 + \lambda_3^2$. In unconstrained uniaxial loading, $\sigma_{22} = \sigma_{33} = 0$. Therefore, the uniaxial compression requires that,

$$\frac{\kappa}{2}J^{5/3}(J-1) = \frac{\mu}{6}(\lambda^2 - J/\lambda) \tag{A8}$$

and

$$P = \sigma_{11} = -\mu\,(\lambda^2 - J/\lambda)/J^{5/3} \tag{A9}$$

where λ is the principal stretch in the direction of the applied load. Using Newton's root-finding method, we solve Eqs. (A8) and (A9) to obtain the uniaxial compression for each layer ($L_i\lambda$) at different applied pressure (*P*), and calculate the resistance ratio using Eq. (A2).